Type-1.5 Superconducting State from an Intrinsic Proximity Effect in Two-Band Superconductors

Egor Babaev[1,2], Johan Carlström[1] and Martin Speight[3]
[1] *Department of Theoretical Physics, The Royal Institute of Technology, Stockholm, SE-10691 Sweden*
[2] *Department of Physics, University of Massachusetts Amherst, MA 01003 USA*
[3] *School of Mathematics, University of Leeds, Leeds LS2 9JT, UK*

We show that in multiband superconductors even small interband proximity effect can lead to a qualitative change in the interaction potential between superconducting vortices by producing long-range intervortex attraction. This type of vortex interaction results in unusual response to low magnetic fields leading to phase separation into domains of a two-component Meissner states and vortex droplets.

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The textbook classification of superconductors, divides them into two classes, according to their behavior in an external field. Type-I superconductors expel low magnetic fields, while elevated fields produce macroscopic normal domains in the interior of the superconductor. Type-II superconductors possess stable vortex excitations which can form a vortex lattice as the energetically preferred state in an applied magnetic field. This picture of type-II superconductivity, as well as the essence of the more complex physics of fluctuating vortex matter, relies on the fact that the interaction between co-directed vortices is purely repulsive. In [1] it was demonstrated that in $U(1) \times U(1)$ superconductors with two independent components, in a wide parameter range, there are vortex solutions which are on the one hand thermodynamically stable, and on the other hand, possess a non-monotonic interaction potential, repulsive at short distances but attractive at larger distances. Long range vortex attraction in the models [1] originates from the circumstance that the coherence length of one of the components is the largest length scale of the problem, and the core of one of the components extends to the region where current and magnetic field (which are responsible for repulsive intervortex interactions) are exponentially suppressed. Indeed such a vortex interaction, along with their demonstrated thermodynamic stability, should cause the system to respond to external fields in an entirely different way from the vortex states of traditional type-II superconductors. Namely, the attraction between vortices should, at low fields, produce the "semi-Meissner state" [1]) featuring (i) formation of voids of vortex-less states, where there are two well developed superconducting components and (ii) vortex clusters where the second component is suppressed by overlapping of vortex outer cores. This kind of external field-induced "phase separation" which, from the point of view of the second component, resembles a mixed state of type-I superconductors, can be interpreted as the system showing aspects of type-I and type-II magnetic response simultaneously. The term type-1.5 superconductivity was coined for this kind of behavior. Note that this magnetic response originates from the existence of three fundamental length scales in the problem (in contrast to the ratio κ of two fundamental length scales which parametrizes single-component Ginzburg-Landau theory), and thus it is entirely different from the inhomogeneous vortex states in single-component superconductors where inhomogeneity can be induced by defects in a type-II superconductor or by tiny attraction caused by various nonuniversal microscopic effects beyond the Ginzburg-Landau theory which might be pronounced in single-component superconductors with κ is extremely close to $1/\sqrt{2}$ [2].

Recently there has been strong and growing interest in multi-band materials where intercomponent interaction can be substantial. Examples are MgB_2 [3, 4] and possibly new iron-based superconductors [5]. The two-band superconductor MgB_2 [3, 4] was regarded in early theoretical and experimental works as a standard type-II superconductor. This was disputed in the recent works by Moshchalkov et al [6, 7], which reported highly inhomogeneous vortex states formation in clean samples in low magnetic fields with vortex clusters (with a preferred intervortex separation scale) and vortex-less Meissner domains strikingly similar to the picture of the semi-Meissner state [1]. In connection with the experiments [6, 7] and recent suggestions that iron pnictides may also be multi-component superconductors, the question arises under what conditions type-1.5 superconductivity is possible (even in principle) in general multi-band systems with a substantial interband coupling.

In this Letter we show that type-1.5 behaviour can arise via a new mechanism in a two-band system with a direct coupling between the bands. The situation which we consider is, in a way, antipodal to that considered in [1]: namely where only one band is truly superconducting while superconductivity in the other band is induced by the interband proximity effect. We address the properties of such a regime by studying the following free energy density (in units where $\hbar = c = m = 1$ and e is the Cooper pair charge).

$$\mathcal{F} = \frac{1}{2}\sum_{i=1,2}|(\nabla + ie\mathbf{A})\psi_i|^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 + \frac{1}{2}\Big(|\psi_1|^2 - 1\Big)^2$$

$$+ \alpha|\psi_2|^2 + \frac{1}{2}\beta|\psi_2|^4 - \eta|\psi_1||\psi_2|\cos(\theta_2-\theta_1) \qquad (1)$$

Here $\psi_{1,2}$ represent the superconducting components associated with two bands. The radical difference with previous studies [1] is that in (1) the effective potential for ψ_2 has only positive terms $\alpha, \beta > 0$, i.e. this band is *above* its critical temperature. It has a nonzero density of Cooper pairs only because of the interband tunneling represented by the term $-\eta|\psi_1||\psi_2|\cos(\theta_2-\theta_1)$ (since the Josephson term favours locked phases we have $\theta_1 = \theta_2 \equiv \theta$). The results can be generalized to including other mixed gradient and density terms in (1). In what follows we will denote the ground state values of $|\psi_1|$ and $|\psi_2|$ by u_1 and u_2. Note that in this model, in general, no explicit expressions for u_1 and u_2 in terms of α, β, η exist, but one can compute power series expansions for them in η,

$$u_1 = 1 + \frac{\eta^2}{8\alpha} + O(\eta^4), \qquad u_2 = \frac{\eta}{2\alpha} + O(\eta^3). \qquad (2)$$

Vortex solutions of the model take the form $\psi_a = \sigma_a(r)e^{i\theta}$, $\mathbf{A} = r^{-1}a(r)(-\sin\theta, \cos\theta)$, where $\sigma_a(\infty) = u_a$ and $a(\infty) = -e^{-1}$. To understand the long-range behaviour of a vortex, we choose gauge so that ψ_1, ψ_2 are real, set $\psi_i = u_i + \chi_i$, and linearize the model about $\boldsymbol{\chi} = (\chi_1, \chi_2)^T = (0,0)^T$, $\mathbf{A} = 0$. The result is a *coupled* Klein-Gordon system with energy density

$$E = \frac{1}{2}\left\{|\nabla\boldsymbol{\chi}|^2 + \boldsymbol{\chi}^T\mathcal{H}\boldsymbol{\chi} + |\nabla\times\mathbf{A}|^2 + e^2(u_1^2+u_2^2)|\mathbf{A}|^2\right\}, \qquad (3)$$

where $\mathcal{H}$ is the Hessian of V about the ground state $|\psi_i| = u_i$, that is, $\mathcal{H}_{ij} = \partial^2 V/\partial|\psi_i|\partial|\psi_j|$. Clearly,

$$\mathcal{H} = \begin{pmatrix} 6u_1^2 - 2 & -\eta \\ -\eta & 6\beta u_2^2 + 2\alpha \end{pmatrix}. \qquad (4)$$

The eigenvalues of $\mathcal{H}$ are the squared masses of the normal modes about the ground state. If $\eta = 0$, then χ_1 and χ_2 decouple and have masses 2 and $\sqrt{2\alpha}$, the first one being in this limit the inverse coherence length of the first condensate, as expected. If $\eta > 0$, both condensates have nonzero ground state values u_1 and u_2 which are not known explicitly, but importantly the normal modes are not χ_1, χ_2, but rather an orthogonal pair $(\chi_1\cos\omega + \chi_2\sin\omega, -\chi_1\sin\omega + \chi_2\cos\omega)$ where $(\cos\omega, \sin\omega)^T$ and $(-\sin\omega, \cos\omega)^T$ are the eigenvectors of $\mathcal{H}$. Physically this means that the recovery of densities in both bands from the core singularity has a strong mutual dependence. The London penetration length is given by the inverse mass of $\mathbf{A}$: $\mu_A = e\sqrt{u_1^2+u_2^2}$. So the linear theory predicts, at large r, the asymptotic formulae

$$\begin{aligned}
|\psi_1| &\sim u_1 - q_1\cos\omega K_0(\mu_1 r) + q_2\sin\omega K_0(\mu_2 r) \\
|\psi_2| &\sim u_2 - q_1\sin\omega K_0(\mu_1 r) - q_2\cos\omega K_0(\mu_2 r) \\
|\mathbf{A}| &\sim r^{-1}(e^{-1} + q_A K_1(\mu_A r))
\end{aligned} \qquad (5)$$

where K_m denotes the m-th modified Bessel function of the second kind, and q_1, q_2, q_A are some unknown real constants depending on α, β, η, e. Recall that $K_m(r) \sim (\pi/2r)^{\frac{1}{2}}e^{-r}$ for all m. This means that, in spite of the presence of two superfluid densities, we *cannot* talk about two distinct coherence lengths (in the GL sense) pertaining to these condensates: the leading term in both $|\psi_1| - u_1$ and $|\psi_2| - u_2$ decays exponentially with the *same* length scale, $\xi = \max\{\mu_1^{-1}, \mu_2^{-1}\}$. At the same time the system retains three fundamenatal length scales, which in this case are the magnetic field penetration length and two inverse masses μ_1^{-1}, μ_2^{-1} of the modes associated with density variation in the coupled bands. Applying the methods of [8], one finds that the asymptotic interaction potential for two well-separated vortices is

$$V \sim 2\pi[q_A^2 K_0(\mu_A r) - q_1^2 K_0(\mu_1 r) - q_2^2 K_0(\mu_2 r)]. \qquad (6)$$

The first term represents repulsion due to current-current and magnetic field interactions, while the last two terms represent attractive forces associated with nontrivial density modulation, mediated in this case by the normal modes, described by scalar fields of mass μ_1 and μ_2. Hence, the linearized theory predicts that vortices should attract at very large separations if $\min\{\mu_1, \mu_2\} < \mu_A$ (and repel if $\min\{\mu_1, \mu_2\} > \mu_A$). It should be emphasized that the above analysis concerns the leading asymptotics of the vortex fields, not the core structure of the vortex directly. As we discuss below, in the presence of interband Josephson tunneling the detailed core structure is principally important for the form of the vortex interaction potential (in contrast to usual single-component superconductors). This core structure cannot be derived in a simple manner from eq. (1). Rather, it must be deduced from numerical solutions of the full, nonlinear field equations which are presented in the second half of the paper.

However for small η we can demonstrate analytically that vortices in the model (1) can attract one another at long range but repel at short range by finding the masses μ_A, μ_1, μ_2. That is, we can find expansions for these masses and the "mixing angle" ω, valid for small η,

$$\begin{aligned}
&\mu_A = e + O(\eta^2);\ \mu_1 = 2 + O(\eta^2);\ \mu_2 = \sqrt{2\alpha} + O(\eta^2) \\
&\omega = \frac{\eta}{|2\alpha - 4|} + O(\eta^2).
\end{aligned} \qquad (7)$$

There is a range of parameters where u_i, μ_i and ω can be computed explicitly, namely $\beta = 0$. Physically, this limit is sensible if $|\psi_2|$ remains everywhere small, which it does for small η, since $u_2 = O(\eta)$. One finds that

$$\begin{aligned}
&u_1 = \sqrt{1 + \frac{\eta^2}{4\alpha}};\ u_2 = \frac{\eta}{2\alpha}u_1;\ \omega = \tan^{-1}\left|\frac{\eta}{2\alpha - \mu_1^2}\right|; \\
&\mu_{1,2}^2 = \frac{p \pm \sqrt{p^2 - 128\alpha^3 - 32\eta^2\alpha^2}}{4\alpha}
\end{aligned} \qquad (8)$$

where $p = 8\alpha + 4\alpha^2 + 3\eta^2$. To understand the intervortex forces at short range we note that for small η, ψ_2 remains

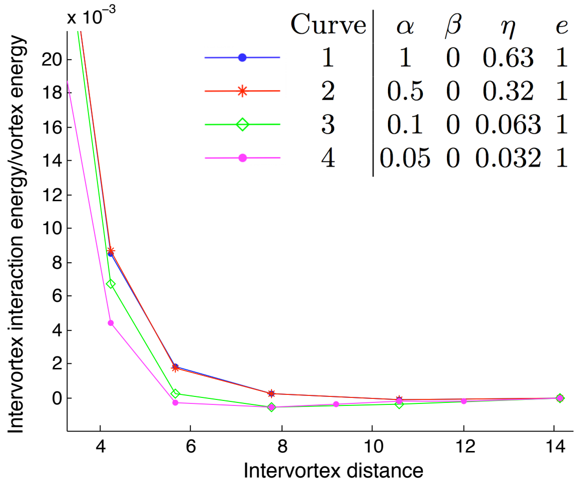

FIG. 1. (Color online) Interaction energy between two vortices as a function of vortex separation in units of $10^{-3}E_v$ where E_v is the single vortex energy for a density ratio of $u_2^2/u_1^2 = 0.1$.

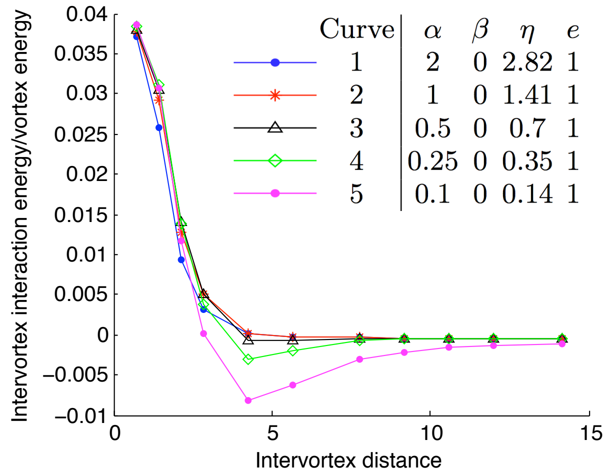

FIG. 2. (Color online) Interaction energy between two vortices as a function of vortex separation for a density ratio of $u_2^2/u_1^2 = 0.5$.

close to zero throughout the core of ψ_1 and in the most of the flux-carrying area, so one expects ψ_2 to contribute negligibly to the interaction energy at short range. In this limit one can approximate the vortex solution at this scale by setting $\psi_2 = 0$ in F yielding a one component GL model with GL parameter $\kappa_{GL} = \sqrt{2}e^{-1}$, leading one to predict short range vortex *repulsion* for $0 < e < 2$ for the effective potential given in eq. (1). So, linear and qualitative analysis suggests that the model (1) does possess type-1.5 superconductivity at least whenever $0 < e < 2$ and the condition for long range attraction holds, namely $\min\{\mu_1^2, \mu_2^2\} < e^2(u_1^2 + u_2^2)$, where μ_i^2 are the eigenvalues of $\mathcal{H}$. In order to test this prediction, and to study regimes where analytic estimates cannot be made, we have performed numerical studies of the model (1) at various parameter values. The computation was conducted as follows: First, two phase windings were created around two fixed points on a numerical grid. Then, the free energy was minimized with respect to all degrees of freedom using a local relaxation method, constrained so that the vortex cores positions remained fixed. The process was then repeated for various separations, yielding an intervortex interaction potential.

First let us consider the regime where the fourth order term in $|\psi_2|$ can be neglected (i.e. $\beta = 0$). In this case we conducted computations with the density ratios $|u_2|^2/|u_1|^2$ being 0.1 and 0.5. The results for intervortex interaction energy are presented in Fig. 1-3. The computed interaction energy is given in units of $2E_v$ where E_v is the energy of an isolated single vortex. The length is given in units of $\sqrt{2}\xi_1$ where ξ_1 is a characteristic constant (the same for all figures) defined as the coherence length which can be associated with this band in the limit of zero coupling to the second band.

In the first case, with density ratio 0.1 we find that in general the density profiles of the condensates can be quite different, even though one of the bands has proximity-induced superconductivity. This can be ascribed to the fact that the mixing angle ω is small (note that $\omega \approx \frac{u_2}{u_1}\frac{2\alpha}{|2\alpha-4|}$) so that the subleading normal mode (of mass $\mu_1 > \mu_2$) dominates ψ_1 at intermediate range. We find that as a consequence of the disparity in the recovery lengths the system crosses over from type-II to type-1.5 behaviour when α and η are sufficiently low (Fig. 1). The low density of ψ_2 means that the attractive part of the interaction is weak. In the curves 3 and 4, we find a slight long range attraction yielding a minimum energy at around the separation of $r = 8$. In the second case (Fig. 2), the density ratio is increased to 0.5. The vortex-vortex binding energy is now much larger, and the minimum energy occurs at a smaller separation. Long range attraction occur in curves 3-5 with a maximum α of 0.5, in contrast to $\alpha \approx 0.1$ in the previous case.

In the third case (Fig. 3), the electric charge has been increased by a factor $\sqrt{2}$, (which is equivalent to decreasing penetration length) which decreases the magnetic repulsion between vortices. Observe the emergence of a new phenomenon: now the energy of an axially symmetric vortex solution with two flux quanta is smaller than the energy of two infinitely separated one-quanta vortices. Nonetheless, the axially-symmetric two-quantum vortex is not stable since the minimum energy occurs at nonzero vortex separation.

Fig. 4 shows the effect of the addition of a fourth order term with coefficient β in the free energy of the proximity-induced component. It demonstrates the persistence of type-1.5 superconductivity when fourth order

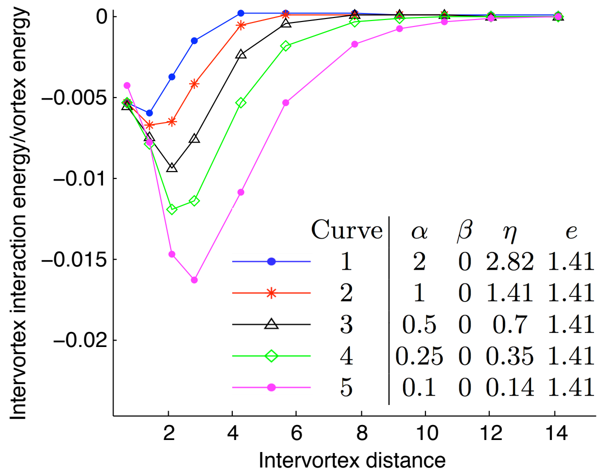

FIG. 3. (Color online) Interaction energy between two vortices as a function of vortex separation for a density ratio of $u_2^2/u_1^2 = 0.5$ for $e = 1.41$.

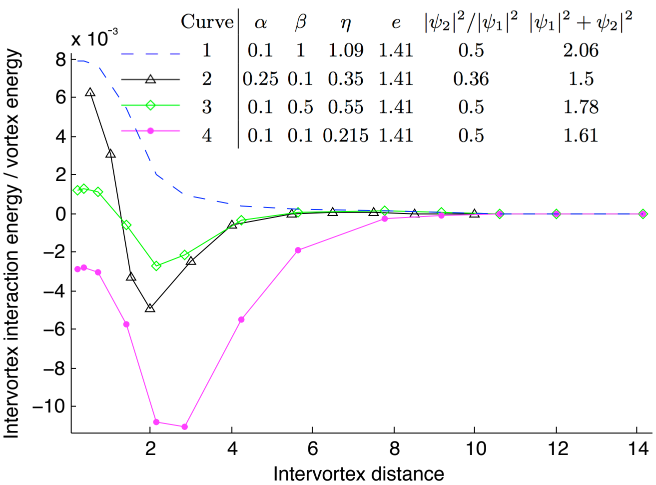

FIG. 4. (Color online) Intervortex interaction in the presence of fourth-order term for ψ_2 in various regimes.

terms are present for ψ_2.

To illustrate the actual behaviour of the fields leading to this unusual intervortex interaction we plot in fig. 5 cross-sections of the density and magnetic field profile corresponding to parameter set 2 in Fig.4. The figure clearly shows that, in spite of the identical long range asymptotics of density behaviour in both bands (as predicted by the linear theory), the rate of density recovery in both bands at *intermediate* scales is actually different.

In conclusion, we considered vortex matter in a situation which can take place in two-band systems: only one band is superconducting while superfluid density is induced in another band via an interband proximity effect. This situation is in a way antipodal to the previously studied unusual vortex interaction arising in condensates with independent coherence lengths [1], e.g. as we showed, the asymptotics of the superfluid densities at large distances from the core in both bands are governed by the same exponential law. However we find that, in contrast to the conventional single-component situation, the presence of even a tiny interband proximity effect can be crucially important. Namely, it gives rise to three fundamental length scales in the problem and to nontrivial variations of the relative superfluid densities in two bands in a vortex producing type-1.5 behaviour in a wide range of parameters. It should manifest itself in the magnetic response which involves a phase separation into vortex and two-component Meissner domains. The effect may be more common near the temperature where the weak band crosses over from active to proximity-induced superconductivity because α should be small near this temperature.

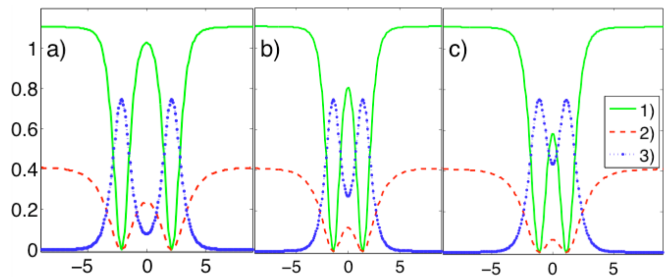

FIG. 5. (Color online) The behaviour of $|\psi_1|$ (curve 1), $|\psi_2|$ (curve 2) and magnetic field (curve 3) for $\alpha = 0.25, \eta = 0.35, \beta = 0.1, e = 1.41$. Separations are (a): 4.24 (corresponding to attractive, $|\psi_2|$-dominated interaction); (b): 2.83 (vicinity of the minimum of the interaction potential); (c): 2.12 (corresponding to domination of repulsive current-current and magnetic interactions).

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